

# Sandpile formation by revolving rivers

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Experimental observation of a new mechanism of sandpile formation is reported. As a steady stream of dry sand is poured onto a horizontal surface, a pile forms which has a thin river of sand on one side flowing from the apex of the pile to the edge of its base. The river rotates about the pile, depositing a new layer of sand with each revolution, thereby growing the pile. For small piles the river is steady and the pile formed is smooth. For larger piles, the river becomes intermittent and the surface of the pile becomes undulating. The frequency of revolution of the river is measured as the pile grows and the results are explained with a simple scaling argument. The essential features of the system that produce the phenomena are discussed.

Sandpiles have received considerable interest because of their intrinsic scientific interest both from the fundamental and applied points of view, and also because they are simple examples of complex systems whose behavior has been used in an attempt to explain a variety of physical, chemical, biological and social phenomena[1]. Conventional understanding of sandpile formation is that as grains of sand are poured onto a horizontal surface, a conical pile develops which grows intermittently through avalanches that “adjust” the angle of repose of the pile about some critical value, or, at least, keep it between two critical values. This mechanism of pile formation has been widely studied in the recent years[2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. Here we report experimental observation of a remarkable new mechanism of pile formation.

Pouring a steady stream of sand into the center of a cylindrical container, as shown in Fig. 1, a pile formed. Then, a continuous river of sand developed flowing from the apex of the pile to the inner boundary of the container. The river, which was narrow compared with the radius of the container, revolved around the pile depositing a helical layer of sand a few grains thick with each revolution. Thus, the pile grew as the river revolved around it. A photograph of a revolving river can be seen in Fig. 2. Within a range of experimental parameters and conditions, the formation of a revolving river was easily reproducible, and very robust. Once formed, a typical river persisted for dozens of full turns around the growing pile, and stopped only when forced to by interrupting the pouring of sand.

In the experiments, a vertical glass tube with a 20 mm inner diameter was initially filled with sand using a funnel. Then a 4 mm hole was opened in the bottom of the tube, allowing sand to fall out of the tube by its own weight. This arrangement produced a steady flow of sand out of the tube at a steady rate of 4.5 g/s for the duration of the experiment. Video cameras recorded both lateral

and top views of the piles during the experiment. (Top views were obtained with the help of a 45° tilted mirror). Two different versions of the experiment were performed, each corresponding to a different boundary condition of the growing pile. In the first version (described above), the pile had a *closed boundary*. The sand was dropped at the center of a cylindrical container, so that the radius of the resulting pile was constant in time. In the second version, the pile had an *open boundary*. No container was present. Instead, the sand fell onto a flat horizontal surface and the radius of the pile increased in time.

Rivers that revolved about the pile in both clockwise and counterclockwise directions were observed. The direction chosen in a particular case depended on the initial conditions. The axial symmetry of the system was therefore spontaneously broken as the river was formed. Viewed from above, the rivers were slightly bent, and always revolved around the pile in the direction of their concavity, as shown in Figs. 2a and 2c. A steady revolving river was typically observed when sand was poured into a container with a 4-6 cm radius. In this case, the surface of the pile was smooth. However, when a container with a radius larger than 6 cm was used, an instability appeared in the flow of the river. The revolving river still developed, growing the pile as before, but the flow of the river was intermittent rather than continuous. In that case, the intermittent flow produced an undulating pattern on the pile surface, visible in Figs. 2c and 2d. The undulating pattern resembles those recently observed for rapid granular flows on an inclined plane[18, 19], but presumably is caused by a different mechanism. The observed pattern was quite regular for containers with a radius just large enough to observe the instability, but became more irregular as the size of the container grew. If a container smaller than 3 cm radius was used, stable revolving rivers were not observed.

The revolving river mechanism of pile formation has also been observed by simply pouring the sand onto a

flat surface. In that case, the crossover from a continuously flowing revolving river, observed in smaller piles, to an intermittently flowing river, observed in larger piles, occurred as the radius of the pile reached about 6 cm. The crossover appeared to be correspond to the pile size needed for the length of the ballistic motion of the sand grains in a river to begin to be damped.

We have varied the drop height in the experiment. For drop heights between 1 cm and 7 cm the results closely follow the description given above. However, for drop heights less than 1 cm or larger than 7 cm, stable rivers were not observed.

The origin of the curved shape of a revolving river and the reason for it moving in the direction of its concavity can be understood by how a river forms. Based on careful observation, revolving rivers appear to form through the following scenario, illustrated in Fig. 3. Initially, sand is poured onto the top of a conical pile and it forms a river flowing straight down one side of the pile (Fig. 3a). Sand begins to build up at the bottom of the river at the edge of the pile, forming a growing inverted V shaped delta of stationary sand (Fig. 3b). The delta grows in size until the river spontaneously chooses to begin to flow down one of the sides of the delta (Fig. 3c). Once it chooses a side, it continues to flow down that side of the delta, depositing sand all along the lower, delta side of the river. As it does so, it rotates about the pile. For smaller piles, the process of rotation was stable. However, for larger piles, it was not. Instead, in that case, a new delta would intermittently begin to form at the bottom of the river (Fig. 3d). When the delta reached sufficient height, the river would “jump” forward in its rotation, and then begin forming yet another new delta.

In order to begin to quantitatively understand the revolving rivers, we measured the time evolution of the angular velocity of river rotation with both closed and open boundaries. As shown in Fig. 4, the angular velocity of river rotation was roughly constant for piles in cylindrical containers, while it decreased in time as  $t^{-\alpha}$ , with  $\alpha = 2/3$ , for open boundary conditions. These results can be explained using the following scaling argument whose geometrical hypotheses are illustrated in Fig. 5. Assume that a new layer of sand is uniformly deposited on a conical pile of radius  $r$  with an angle of repose  $\theta_c$ , and that the volume of sand added per unit time is  $F$ . For a system with a closed boundary, the thickness of an added layer is proportional to  $\delta h$  (see Fig. 5a). Therefore, the volume of sand deposited in each rotation of the river

$$V = \frac{\pi r^2 \delta h}{\cos \theta_c}$$

is constant in time. The angular velocity of the river

$$\omega = 2\pi \frac{F}{V} \quad (1)$$

is therefore also constant in time. In our experiments, we measured  $\delta h = 2$  mm,  $\phi_c = 33^\circ$ , and  $F = 0.35$  cm<sup>3</sup>/s. However, for a system with an open boundary the radius of pile grows in time. In this case, the thickness of each layer is proportional to  $\delta r$  (see Fig. 5b). The volume of sand deposited in a rotation of the river is

$$V = \pi \tan \theta_c r^2 \delta r \quad (2)$$

where  $r$  is a function of time, but  $\delta r = \delta h / \sin \theta_c$  is constant. Thus, from this result and Eqn. 1,  $\omega \sim r^{-2}$ . The pile radius increases at a rate of

$$\frac{dr}{dt} = \omega \delta r$$

Integrating this expression, we get  $r \sim t^{1/3}$ , and therefore

$$\omega \sim t^{-2/3}.$$

Our scaling argument matches well the experimental results shown in Fig. 4a. In the case of Fig. 4b, although this argument correctly predicts the scaling of the experimental data for larger piles, it does not properly describe the behavior of smaller piles, presumably due to the fact that our geometrical assumptions are inaccurate near the tip of the pile.

The appearance of revolving rivers is quite sensitive to the type of sand used in the experiments. In the results reported here, sand from Santa Teresa, Cuba, was used. It consists in irregularly shaped grains of size 30–250  $\mu m$  made of almost pure silicon oxide. It was also quite dry. Revolving rivers were still observed if the sand was meshed to remove grains smaller than 90  $\mu m$  and larger than 160  $\mu m$ . However, other sands from Cuba, USA, Norway and Tunisia were tried, including ones high in Calcium Carbonate, and ones high in Magnetite, but no revolving rivers were observed within our experimental conditions (a river occasionally formed in those sands, but it disappeared in fractions of a second). Revolving rivers also were not observed if glass beads having roughly the same size as the Santa Teresa sand were used. It is therefore suspected that the effective coefficient of friction between grains, and the mass density of grains may be important factors determining if revolving rivers appear in the formation of piles. These elements must be included in a future “first principles” model of the revolving rivers.

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## Figure Captions

Fig. 1. Experimental setup

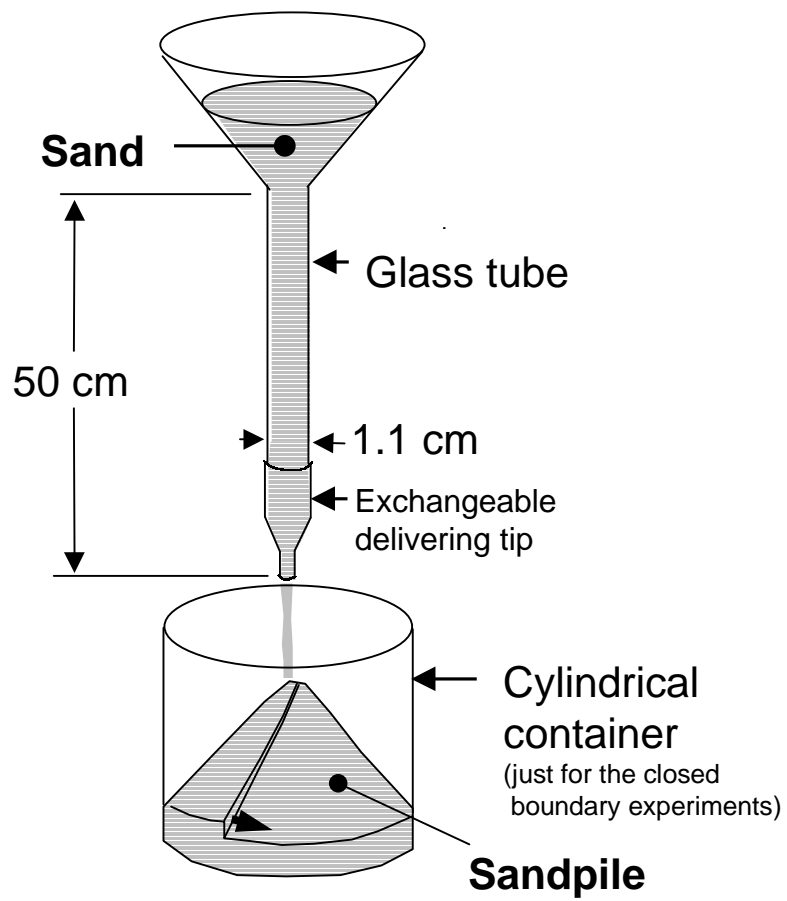
Fig. 2. Formation of a pile of sand by revolving rivers. The sand is poured vertically on the center of cylindrical containers with flat, horizontal bottoms at a deposition rate of  $0.35 \text{ cm}^3/\text{s}$ , from a constant height of 1.5 cm above the apex of the pile. (a) Top view of a pile growing into a 5 cm radius container, where the continuous river can be identified. (b) Lateral view of the pile shown in (a). (c) Top view of a pile growing into a 10 cm radius container where an intermittent river and the related pattern can be identified. (d) Lateral view of the pile shown in (c) (the photo shows about 3 cm of the container's perimeter). In all cases, arrows indicate the revolving direction.

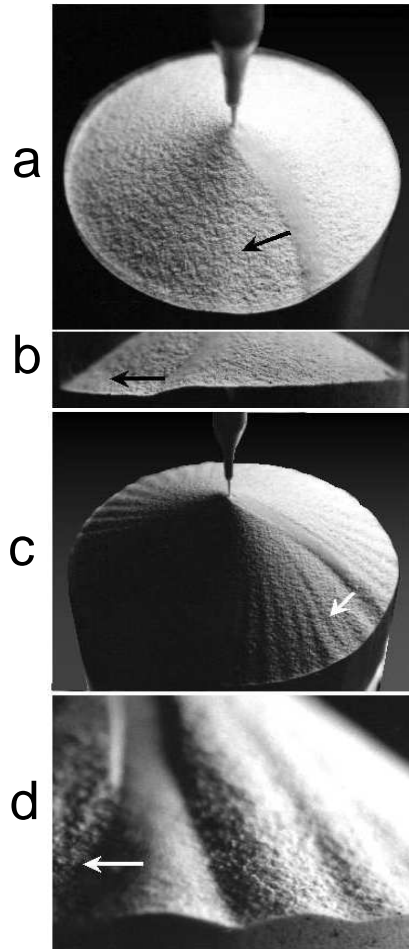
Fig. 3. Development of a revolving river. (a) A river

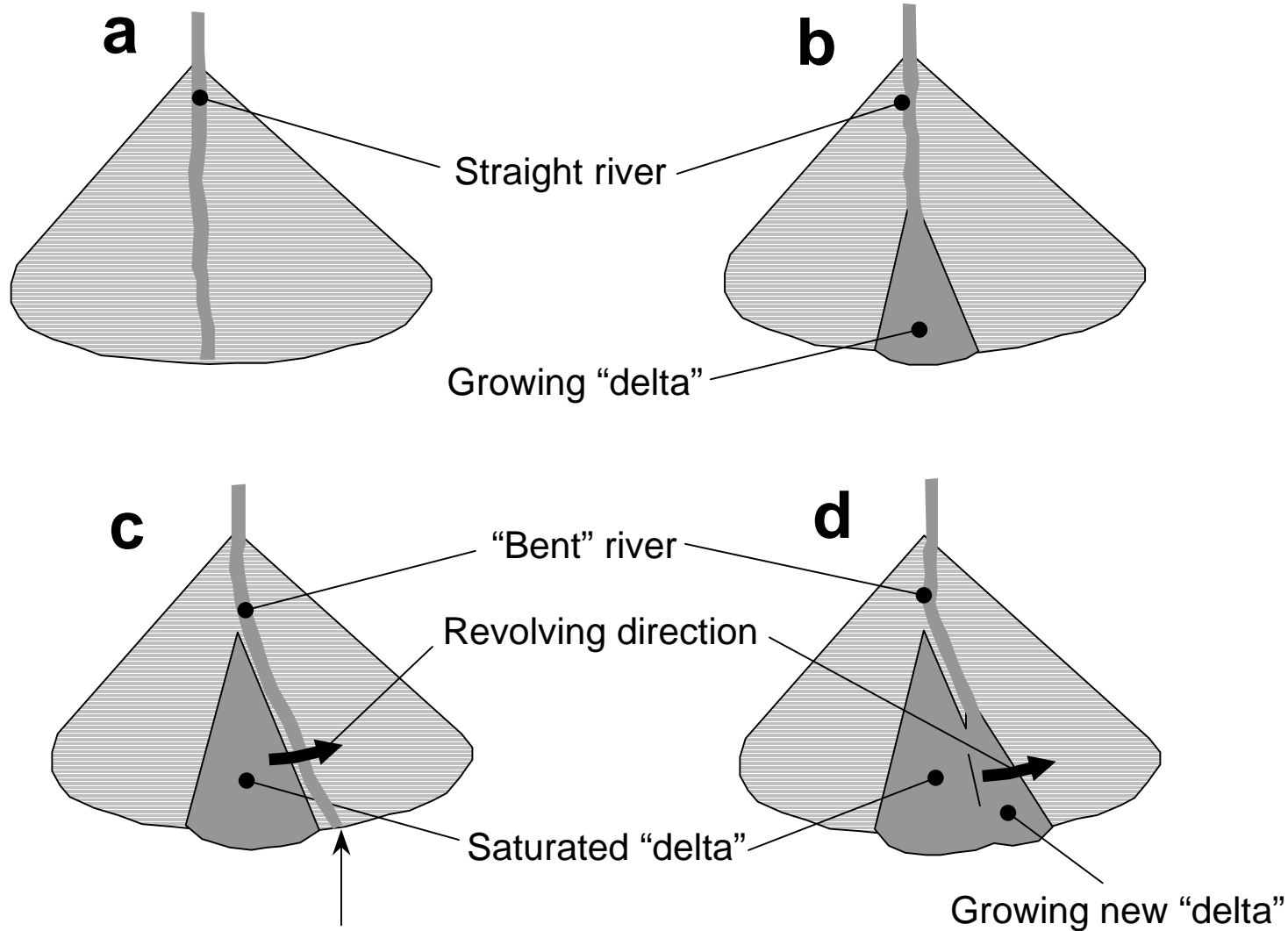
flows straight down the side of the pile, and a delta begins to form at its bottom. (b) The delta continues to grow. (c) When the delta is sufficient size, the river begins to flow down one side and rotate around the pile. (d) If the pile is sufficiently large, a new delta forms intermittently at the bottom of the river, causing the rotation of the river to become intermittent.

Fig. 4. Time dependence of the angular speed of revolving rivers for (a) closed boundary conditions in the continuous regime (5 cm-radius container) and (b) open boundary conditions. The solid line in (b) has a slope of  $-2/3$ .

Fig. 5. Geometrical hypotheses of our scaling argument for (a) closed boundary conditions and (b) open boundary conditions.







New "delta" begins here for larger piles

